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APPLICATION OF DYNAMIC PROGRAMMING TO THE CULLING DECISION IN DAIRY CATTLE

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INTRODUCTION

Selection and culling are the two procedures of herd improvement. Because of the power of selection through artificial breeding, herd improvers have paid particular attention to measuring the genetic value of yield and now there is worldwide adoption of the methods of assessing genetic value proposed by Johanson and Robertson (1953) and first applied by the Milk Marketing Board of England and Wales (McArthur, 1954).

However, using genetic value as a criterion for culling an arbitrary proportion of low genetic valued cows may not be the best way of using herd test information to maximize profits. Dynamic programming is used in industry for optimum replacement decisions (Wagner, 1970), and a dynamic programming approach for culling dairy cows has been published by Smith (1972). It has the advantage of being able to incorporate economic variables into the decision as to whether a cow should be culled or not, and to assess the long-run consequences of keeping or replacing a cow.

The purpose of the research reported here was to design a stochastic dynamic programming model suitable for culling decisions in a New Zealand herd-tested factory-supply herd, breeding A.B. Jersey replacements. The culling decision rules derived by this model have been evaluated by comparing the resulting profits with a herd culled on genetic value.

THE DYNAMIC PROGRAMMING APPROACH

The basic elements of the stochastic dynamic programming approach are illustrated with the decision-probability tree in Fig. 1. (The appendix gives the formal dynamic recursive relationship.) In this tree, three possible record states are illustrated — 400 lb, 300 lb and 200 lb of M.E. milkfat. In practice there were 80 production states. There are also two other states — "die" and "fail", the latter referring to enforced culling resulting from infertility or old age. Both "die" and "fail" lead to enforced replacement.

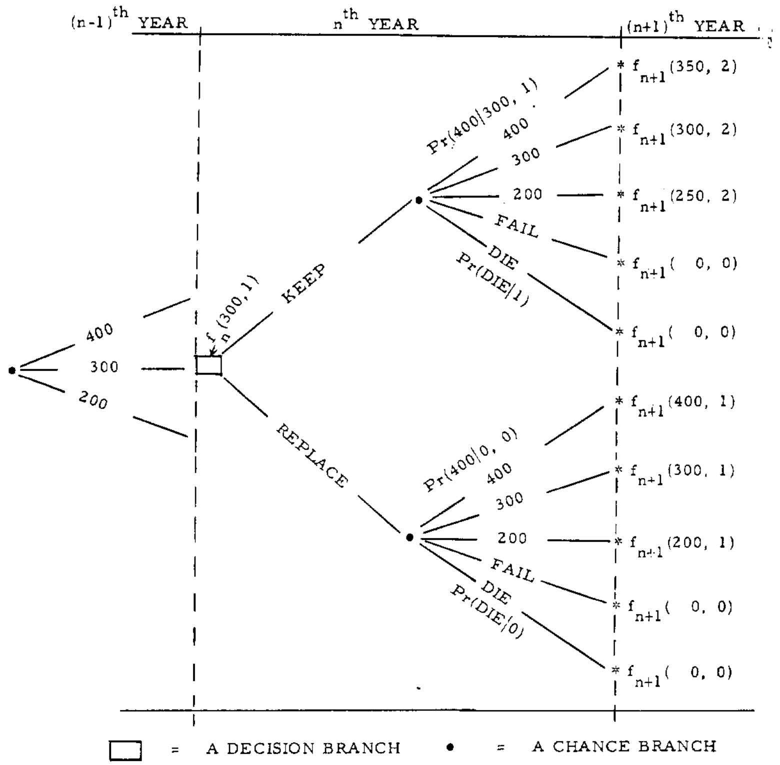

FIG. 1: *A decision-probability tree for the* nth *year.*

Figure 1 is divided into this year (the nth year), last year (the $(n-1)$th), and next year (the $(n+1)$th). The diagram shows a cow which last year produced 300 lb fat. On the basis of this single record a decision must be made at the beginning of the nth year whether to keep or replace her. The "keep" decision branches by chance into one of the five states just described. Each of these states has a certain probability of occurring given that the cow has a 300 lb record based on one lactation. Following up the "replace" branch there are the same five states but here the probabilities will be different — these being the probability of, say, 300 lb fat, given no previous record and based on zero lactations.

At the tips of the branches running into the next year are the symbols f_{n+1} (R, k). These mean the value in dollars of following the best replacement policy in subsequent years for a cow with an average record of R based on k records. At the tip of the 400 lb branch is the symbol f_{n+1} (350, 2), meaning the value of a cow averaging $(300 + 400)/2 = 350$ lb over 2 records. At the tips of the "die" and "fail" branches are the symbols f_{n+1} (0, 0) referring to the value of a replacement cow (with no records) when the best decisions are made over future years about her replacement.

In this year (the *n*th year), the value of following the best replacement policy for a cow with a 300 lb record based on 1 record (f_n (300, 1)) can be found by taking expected values of the "keep" and "replace" branches. The higher of the two values becomes f_n (300, 1).

Values f_{n+1} (R, k) are found by starting in the future at a time when the herd will be sold up and working back year by year. In this study the recursive calculations were rolled back until a stable set of culling decisions was reached for all combinations of "average record levels" and "numbers of records". It usually took about 15 years to reach this stable state. The culling policy derived for each "record level" — "number of records" combination is optimum for planning horizons for 15 years to infinity and near-optimum for shorter horizons.

OPTIMUM CULLING POLICIES

Optimum culling policies were derived by dynamic programming for 4 sets of economic conditions.

High Dairy Prices — High Beef Prices ($High_D$ — $High_B$)
High Dairy Prices — Low Beef Prices ($High_D$ — Low_B)
Low Dairy Prices — High Beef Prices (Low_D — $High_B$)
Low Dairy Prices — Low Beef Prices (Low_D — Low_B)

The details of these prices are shown in Table 1.
Other parameters used in the model are given in Table 2.

TABLE 1: PRICE LEVELS FOR ECONOMIC CONDITIONS

	Price of Milkfat per lb ($)	*Value of a Cull Cow* ($)	*Cost of a Replacement Heifer* ($)
$High_D$ — $High_B$	0.40	80	130
$High_D$ — Low_B	0.40	40	90
Low_D — $High_B$	0.30	80	110
Low_D — Low_B	0.30	40	70

TABLE 2: PARAMETERS USED IN D.P. MODEL

Standard deviation of first lactation records	65 lb
Repeatability	0.60
Expected mature equivalent production level of replacements	370 lb
Age correction, first lactation	76 lb
Age correction, second lactation	43 lb
Age correction, third lactation	10 lb
Discount factor for 1 year	0.909

Figure 2 shows the culling policy for two of the four conditions derived by dynamic programming. The graph line indicates the upper level (M.E.) at which cows should be culled. The Low_D — Low_B policy was identical with the Low_D — High_B policy shown, and the High_D — Low_B was almost identical with the High_D — High_B policy. The small difference between the two lines shows the insensitivity of the culling decision to prices. The Low_D — High_B policy indicates more stringent culling when dairy prices are low. This increases the production of cull cow beef by rearing more replacements and culling out the low producers. However, the swing to this policy should not be very marked.

The initial fall of the culling decision line is due to the lower performance of second and third calvers, and the subsequent rise is due to the increasing repeatability of multiple records.

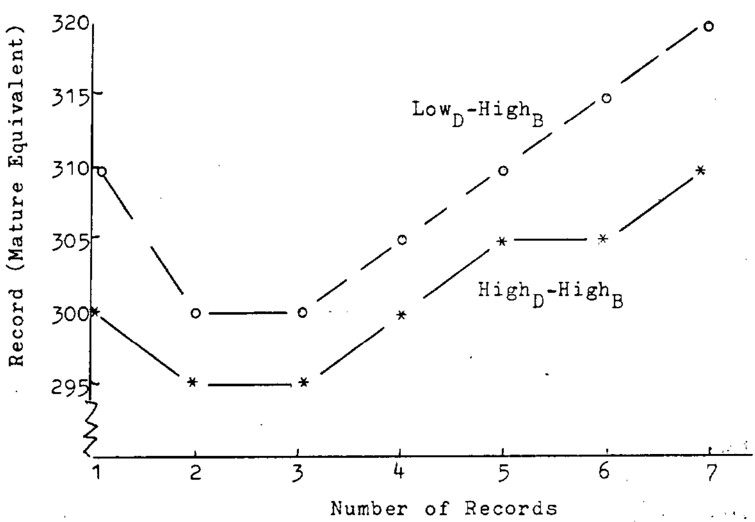

FIG. 2: *Upper culling decision lines for optimum policy.*

EVALUATION OF OPTIMAL POLICY

A Monte Carlo simulation computer program was written which generated records for a typical mixed-aged herd of 100 cows with an average M.E. performance of 330 lb fat which was then subjected to a normal 20% replacement rate with the cows of lowest genetic value for production being replaced. This amounted to approximately the worst 5% on merit being removed, when enforced replacement because of death and failure had been carried out.

This was the normal benchmark against which the dynamic programming culling rules were run over a 20-year period. Being a stochastically simulated situation, the difference in favour of the dynamic programming rules varied from one run to the next. Five 20-year runs were made for each combination of prices. Table 3 shows the differences in mean present value and their standard errors.

TABLE 3: IMPROVEMENT IN PRESENT VALUE WITH D.P. CULLING RULES
($N = 5$)

	Mean	*Standard Error*
High-High	\$221	\$195
High-Low	\$277	\$314
Low-High	\$201	\$182
Low-Low	\$193	\$175

The differences in mean present value were not significant in the statistical sense. If these differences ar taken as the best estimates of the true differences, then a present value difference of about \$200 over 20 years is equivalent to only about 23 cents a cow per year.

As anticipated from the culling decision line, the culling rate was slightly heavier using dynamic programming when prices of milkfat were low — the culling for production being 6.8% instead of 5.2% for low milkfat prices and 5.5% under the conventional 20% replacement rate. It is interesting that replacement rates adopted by farmers are so close to the optimum rate.

CONCLUSION

Optimizing culling rules derived by dynamic programming to maximize profit do not increase the gains from culling appreciably and hence culling rules based on dynamic programming are unlikely to supersede genetic value as a criterion for culling dairy cows.

REFERENCES

Johanson, I.; Robertson, Alan, 1953: *Proc. Brit. Soc. Anim. Prod.*: 79.
McArthur, A. T. G., 1954: *Proc. Brit. Soc. Anim. Prod.*: 75.
Smith, B. J., 1971: *Univ. Florida Agric. Exp. Stn. Bull. 745.*
Wagner, Harvey, 1969: *Principles of Operations Research.* Prentice Hall.

APPENDIX

The dynamic recursive relationship for determining the optimum culling policy is:

$f_n\{R_i, l\} = \text{Max.}$

Either

$$\Big(\sum_{j=1}^{80} Pr(R_j \mid R_i, l)[(R_j - A_{l+1})V + \alpha f_{n+1}\{(lR_i + R_j)/(l+1), (l+1)\}]\Big)$$
$$\times (1 - Pr(F_{l+1}) - Pr(D_{l+1})) + Pr(F_{l+1})\phi(R_i, l)V + \alpha f_{n+1}\{0,0\} - Pr(D_{l+1})$$
$$\times \alpha(f_{n+1}\{0,0\} - B).$$

Or

$$\Big(\sum_{j=1}^{80} Pr(R_j \mid 0,0)[(R_j - A_1)V + \alpha f_{n+1}\{R_j, 1\}] - C + B\Big)$$
$$\times (1 - Pr(F_1) - Pr(D_1)) + Pr(F_1)\phi(0,0)V + \alpha f_1\{0,0\})$$
$$+ Pr(D_1)\alpha(f_1\{0,0\} - B).$$

$f_n\{R_i, l\}$ is the value of the optimal culling policy for a cow with an average record of R_i made up of l records. The continuous variable R was converted to 80 discrete levels at 5 lb of milkfat intervals. $R_1 = 130$ lb, $R_2 = 135$ lb, . . . , $R_{80} = 525$ lb.

$Pr(R_j|R_i, l)$ is the probability of the jth record level occurring (R_j) given an average record level of R_i based on l records.

$(R_j - A_{l+1})V$ is the value of the jth record in dollars. R_j is the jth record expressed as a mature equivalent figure so that A_{l+1}, the age correction factor for an animal with $(l + 1)$ records must be subtracted before multiplying by V which is the price per pound of milkfat in dollars.

$\alpha f_{n+1}\{(lR_i + R_j)/(l + 1), (l + 1)\}$ is the discounted value in dollars of the optimal replacement policy of a cow in the $(n + 1)$th year over subsequent years with an average record of $(lR_i + R_j)/(l + 1)$ based on $(l + 1)$ records. α is the discount factor for allowing for delay in the arrival of benefits of one year and is equal to $1/(1 + q)$ where q is the rate of interest as a proportion.

$Pr(F_{l+1})$ and $Pr(D_{l+1})$ is the probability of a cow failing and dying respectively given she is making the next record — the $(l + 1)$th record. The probabilities were derived from the wastage frequency information published by the N.Z. Dairy Board in their 1955-6 report. The wastage headings of accident, calving, tuberculosis, bloat, metabolic, and death were assumed to imply death. Cows dying were assumed to produce no milkfat during the season. Old age, sterility, and mastitis were the headings which implied failure. Failing cows were assumed to produce their expected record based on past performance.

$\phi(R_i,l)$ is the expected production in pounds, a function of the cow's average record (R_i) consisting of l records.
$\phi(R_i,l) = (lr/(1+(l-1)r))(R_i - H) + H - A_{l+1}$,
$lr/(1+(1-r)r)$ being the repeatability of l records and H being the base herd average — 330 lb of milkfat was used in this study.

$\alpha f_{n+1}\{0,0\}$ is the discounted value of a replacement heifer. These replacements had an assumed expected M.E. level of 370 lb fat with a standard deviation of 65 lb fat. In the case of a cow that dies there is no cull cow return so the discounted value of the future optimal policy is $\alpha(f_{n+1}\{0,0\} - B)$. In the case of the expression for the "replace" alternative, the symbols used have already been explained except that $\phi\{0,0\}$ stands for the expected production of an A.B. replacement, C is the cost of a replacement and B is the value of a cull.

At the end of the planning horizon — the nth year — salvage values of cows of all ages are provided on the assumption that all cows will be sold — the younger cows being worth more.
$f_N(R_i,l) = ((C-B)/8)(8-l) + B$ for $l = 1,2, \ldots 8$
The probability distribution $Pr(R_j/R_i,l)$, $j = 1,2, \ldots 80$ was calculated as follows.
Let σ be the standard deviation of records and r_l, the repeatability of l records,
$r_l = lr/(1+(l-1)r)$.
The standard deviation of records given l previous records (σ_l) is
$\sigma_l = \sqrt{[\sigma(1-r_l^2)]}$
The mean of the probability distribution given an average record of R_i based on l record $(\mu|_{R_i,l})$ is

$$\mu|_{R_i,l} = (R_i - H)r_l + H.$$

Assuming a discrete approximation to a normal distribution,

$$Pr(R_j|R_i,l) = Pr(Z_j)$$

where

$$Z_j = (R_j - \mu|_{R_i,l})/\sigma$$

$$Pr(Z_j) = e^{-Z_j^2/2} \Big/ \sum_{j=1}^{80} e^{-Z_j^2/2}$$

Copies of the computer program used to make these calculations are available from the author.